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SHORT COMMUNICATIONS

PASSIVE PRESSURE DETERMINATION BY METHOD OF SLICES

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SUMMARY

A method of slices satisfying all the conditions of statical equilibrium has been developed to deal with the problem of determination of passive earth pressure over a retaining wall in sand. A method similar to that of Morgenstern and Price,[1] which was used to solve the stability of slopes, has been followed. The earth pressure coefficients with the proposed methodology have been computed for a vertical retaining wall for both positive and negative wall friction angle. Also examined is the variation of the interslice shear force between the retaining wall and the Rankine Passive boundary. Due to complete satisfaction of the equilibrium conditions, the method generates exactly the same earth pressure coefficients as computed by using Terzaghi's overall limit equilibrium approach. © 1997 by John Wiley & Sons, Ltd.

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Key words: method of slices; sands; stability; retaining walls; passive pressure

INTRODUCTION

Method of slices is widely used for solving the slope stability problems.[2,3] The method, in addition, has also been employed to solve the earth pressure problems.[4,5] However, in either case, the problem is statically indeterminate in nature. Assumptions generally dealing with the interslice forces are normally being made to solve the problems. Fellenius[2] assumed that the resultant of the interslice forces acts in a direction parallel to the base of each slice. In the Bishop method,[6] which is widely used at present, it is being assumed that the resultant of the interslice forces acts in a horizontal direction. The point of the application of the resultant interslice force is assumed in the Janbu method.[7] The method of Morgenstern and Price,[1] is based upon the assumption of the distribution of interslice forces taken in such a manner that the conditions of statical equilibrium are completely satisfied. What is presented here is a method which is similar to the method of Morgenstern and Price,[1] but deals with the determination of passive earth pressure and the variation of interslice shear forces over a vertical retaining wall in a cohesionless medium for both positive and negative wall friction angles.

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INDETERMINACY OF THE SYSTEM IN THE METHOD OF SLICES

In order to compare the earth pressure problem with stability of slopes, the number of unknowns and equations of equilibrium are examined as a consequence of discretizing the soil mass contained within the periphery of the potential failure surface into a number of slices. The comparison is shown in Table I. It can be seen that the degree of indeterminacy of the earth pressure problem is higher by one degree as compared to that of the problem of stability of slopes. However, if the point of application of the resultant earth pressure over the back of the retaining wall is suitably assumed, then the degree of the indeterminacy in both the problems becomes equal to $(2n-2)$. The width of the slice can be chosen very small as compared to its height. Therefore, the normal force, N_i, can be assumed to act at the mid-point of the base of each slice. This approximation results in 'n' number of assumptions and consequently the resultant degree of indeterminacy of the system becomes equal to $(n-2)$.

Table I. Unknowns and equations of equilibrium in the method of slice

Unknown Parameter	Total number of unknowns		Remarks
	Slope stability problem	Earth pressure problem	
(a) Force N_i normal to the base of each slice	n	n	—
(b) Force P_i normal to the interface of slices	$(n-1)$	n	In the case of slope stability problem, P_i on the boundary faces will be known whereas in the case of earth pressure problem this force on the wall is required to be determined
(c) Shear force T_i on each interface between slices	$(n-1)$	$(n-1)$	In the case of slope stability problem, T_i on the boundary face will be known whereas in the case of earth pressure problem this force on the wall will also become a known quantity by using P_i and prescribed value of δ
(d) Factor of safety	1	0	In the case of earth pressure problem, factor or safety as defined in the slope stability problem, will always be equal to one
(e) Point of application a_i of force N_i normal to the base of each slice	n	n	—
(e) Point of application z_i of force P_i normal to the interface of slices	$(n-1)$	n	In the case of slope stability problem, this parameter will be known on the boundary faces, whereas in earth pressure problem the point of application of the resultant earth pressure on the wall is an unknown quantity
Total number of unknowns	$5n-2$	$5n-1$	
Total number of available equations of equilibrium	$3n$	$3n$	For each slice three conditions of equilibrium are available
Degree of indeterminacy	$2n-2$	$2n-1$	

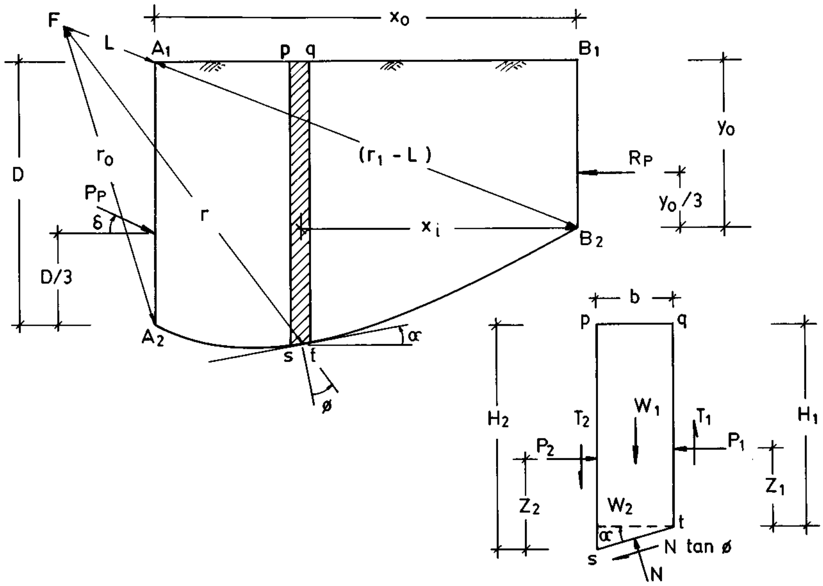

Figure 1. Failure surface and free body diagram of a slice in positive δ

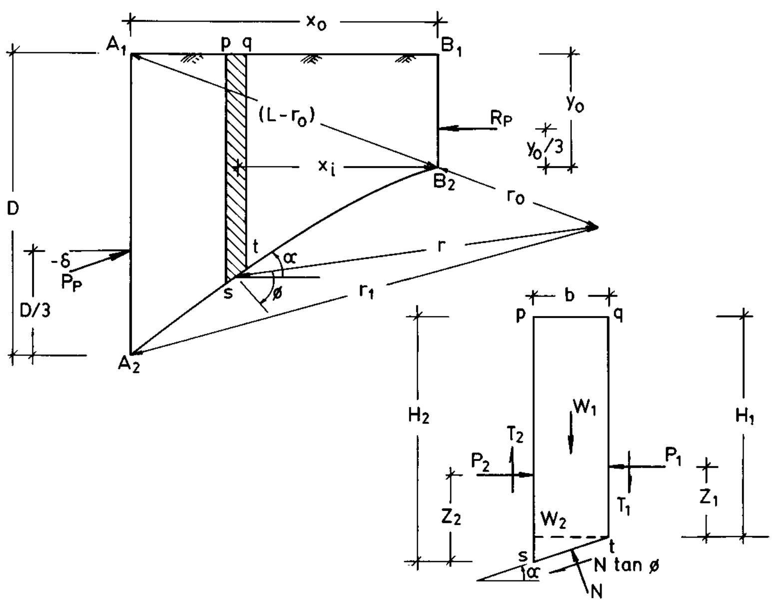

Figure 2. Failure surface and free body diagram of a slice in negative δ

FAILURE SURFACE

In the case of the problem of passive earth pressure on a vertical retaining wall, depending on the nature of the values of wall friction angle, δ, two different natures of curvature of the lower portion of the failure surface generate in the soil mass as shown in Figure 1 for positive values of δ and in Figure 2 for negative values of δ. The value of δ is taken as positive when the wall moves

relatively in a downward direction as compared to the soil mass. From the known values of the orientation of the wall and its surface friction angle, the nature of the lower portion of the failure surface, whether convex or concave, can be easily determined.[8] To facilitate comparison of the proposed method with the overall limit equilibrium approach as given by Terzaghi,[9] the shape of the failure surface has been taken as a combination of a straight line near the ground surface, in the Rankine passive zone, and an arc of the logarithmic spiral near the bottom of the retaining wall as shown in Figures 1 and 2.

BOUNDARY CONDITIONS

For soil wedge $A_1B_1B_2A_2$ (Figure 1), the following boundary conditions should be satisfied:

(1) Along the Rankine passive boundary B_1B_2, the magnitude of the shear force shall be equal to zero, and the normal force R_p should act at a height $y_0/3$ and its magnitude shall be equal to $\gamma K_{pr} y_0^2/2$; in which, $K_{pr} = \tan^2(\pi/4 + \phi/2)$, Y_0 equals distance B_1B_2, and γ equals the unit weight of the soil.
(2) Along the ground surface A_1B_1, the magnitude and the distribution of the stresses shall be as specified. For the problem in question, these stresses are equal to zero.
(3) All along the length of the failure surface, Mohr–Coulomb's failure criterion, shall be satisfied.
(4) Along the retaining wall, the resultant passive earth pressure shall make an angle δ with the normal to the wall. Also, it is being assumed that the resultant earth pressure acts at the bottom one-third height of the wall.

DISTRIBUTION OF THE INTERSLICE SHEAR FORCE

As is seen earlier, the present problem is statically indeterminate with its degree of indeterminacy being equal to $(n - 2)$. The problem has been made determinate by establishing the distribution of the interslice shear force in such a manner, as given below, so that all the conditions of statical equilibrium are satisfied.

Let P_i be the normal force on any ith slice interface; T_i the shear force on any ith slice interface; and δ_i the average interslice friction angle for any ith slice interface. Then

$$T_i = P_i \tan \delta_i \tag{1}$$

If x_i is equal to the horizontal distance of any ith slice interface from the Rankine boundary B_1B_2, then from the boundary condition (1) and (4): For

$$\begin{aligned} x_i = 0; &\quad \delta_i = 0 \\ x_i = x_0; &\quad \delta_i = \delta \end{aligned} \tag{2}$$

in which x_0 is the horizontal distance between the retaining wall and the Rankine passive boundary B_1B_2.

The distriution of the slice interface friction angle, δ_i, has been chosen to be of the following form:

$$\delta_i = C_0 x_i^F + D_0 \tag{3}$$

In which, C_0, D_0 and F are three unknowns, the values of which have been determined by satisfying the available boundary conditions in the following manner:

By making use of equation (2),

$$D_0 = 0$$

$$C_0 = \delta/[x_0]^F$$

Substituting these values of C_0 and D_0 in equation (3),

$$\delta_i = \delta[x_i/x_0]^F \tag{4}$$

The value of the remaining unknown, i.e., the factor F, will then be found out on the basis of the condition that the resultant earth pressure acts at the bottom one-third height of the wall.

EQUILIBRIUM CONDITIONS

Consider any slice *pqts* as shown in Figure 2.

Let W be the total weight of the slice, W_1 the weight of rectangular portion of the slice, and $W_1 = \gamma H_1 b$, W_2 the weight of triangular portion of the slice, and $W_2 = \gamma(H_2 - H_1)b/2$; $W = W_1 + W_2$, H_1, H_2 are the heights of vertical faces *qt* and *ps*, b the width *pq*, α the inclination of the base of the slice with the horizontal, x_1, x_2 the horizontal distances of the vertical faces *qt* and *ps* from the Rankine passive boundary B_1B_2, T_1, T_2 the shear forces acting on the sides *qt* and *ps*, P_1, P_2 the normal forces acting on the sides *qt* and *ps*, Z_1, Z_2 the vertical distances between the points of application of forces P_1 and P_2 from the base of the slice (see Figure 1); and N the normal force on the base.

1. *Vertical equilibrium condition*

From the vertical equilibrium condition of the slice, it can be shown that

$$N = \frac{[W + (T_2 - T_1)]}{(\cos\alpha - \tan\phi \sin\alpha)} \tag{5}$$

2. *Horizontal equilibrium condition*

By considering the horizontal equilibrium condition, it can be seen that

$$N = \frac{(P_2 - P_1)}{(\tan\phi \cos\alpha + \sin\alpha)} \tag{6}$$

Equating equations (5) and (6),

$$P_2 = P_1 + (W + T_2 - T_1)\, f(\alpha, \phi) \tag{7}$$

in which,

$$f(\alpha, \phi) = \frac{(\sin\alpha + \tan\phi \cos\alpha)}{(\cos\alpha - \tan\phi \sin\alpha)}$$

Combining equations (1) and (4),

$$T_2 = P_2 \tan\left[\delta\left(\frac{x_2}{x_0}\right)^F\right] \tag{8}$$

Substituting the above value of T_2 in equation (7),

$$P_2 = \frac{[P_1 + (W - T_1)\, f(\alpha, \phi)]}{\{1 - \tan[\delta(x_2/x_0)^F]\, f(\alpha, \phi)\}} \tag{9}$$

3. *Moment equilibrium condition*
By taking the moment of all the forces acting on the slice about its base centre.

$$Z_2 = \frac{[P_1(Z_1 + b/2\tan\alpha) + P_2 b/2\tan\alpha + (T_1 + T_2)b/2 + W_2 b/6]}{P_2} \tag{10}$$

SOLUTION PROCEDURE

After dividing the soil mass into a number of slices, a certain value of the factor F, which is unknown, is chosen. By making use of the above equations, the computations have been performed for all the slices starting from the Rankine passive boundary, and the point of application of the resultant earth pressure on the wall is obtained. If, its location is different from the bottom one-third height of the retaining wall, the value of F is then altered, and the above procedure is repeated again. The process is continued till the correct position of the point of application of the resultant earth pressure on the wall is finally obtained. In this way, for any chosen failure surface, the magnitude of the earth pressure by the method of slices is finally obtained. Further, by trying a number of different positions of the failure surface, the minimum value of the required passive earth pressure has been determined.

RESULTS

For performing the computations, the number of slices in all the cases is taken equal to hundred as the convergence was found to have been achieved. The value of ϕ was varied from 10 to 50°, and δ values were varied between $-\phi$ to $+\phi$.

Passive earth pressure coefficients

Defining the earth pressure coefficient, K_p, in the following manner:

$$K_p = P_p \cos(\delta)/(D^2/2) \tag{11}$$

The computed values of the passive earth pressure coefficients by the proposed method of slices, and by the Terzaghi's limit equilibrium method are exactly the same. This is only to be expected since all the conditions of equilibrium are satisfied for all the slices, and also the point of application of the resultant earth pressure lies at the bottom one-third of the retaining wall. Figure 3 shows the variation of K_p with δ/ϕ.

Factor F

The variation of the factor F with respect to ϕ and δ/ϕ has been shown in Figure 4. It can be seen that δ/ϕ is having a considerable effect on the values of the factor F. The value of F increases continuously with increase in δ/ϕ. Comparatively, the values of δ close to zero, it was seen that even large variations in the values of F do not seriously affect the location of the point of application of resultant earth pressure on the wall, as a result of which, the values of F in such cases were interpolated.

Variation of δ_i

The manner in which the slice interface wall friction angle δ_i, changes from the Rankine passive boundary B_1B_2, to the retaining wall A_1A_2 has been shown in Figure 5 for a typical value of

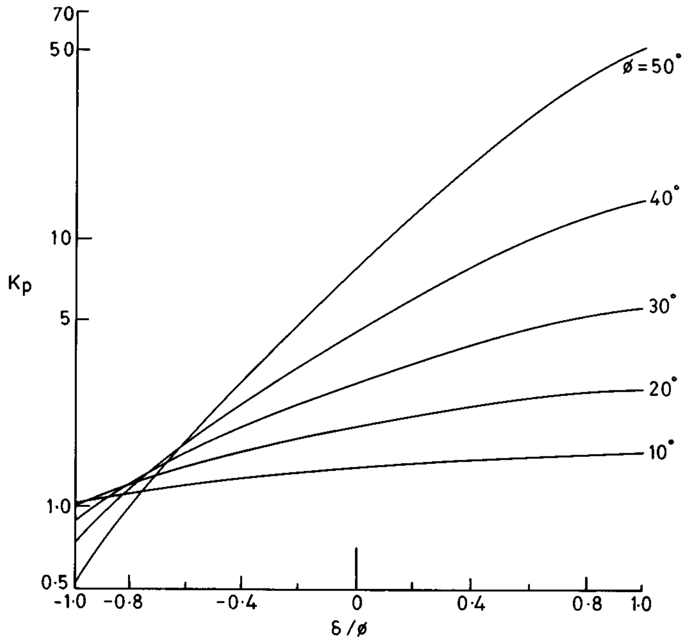

Figure 3. Variation of the passive earth pressure coefficients

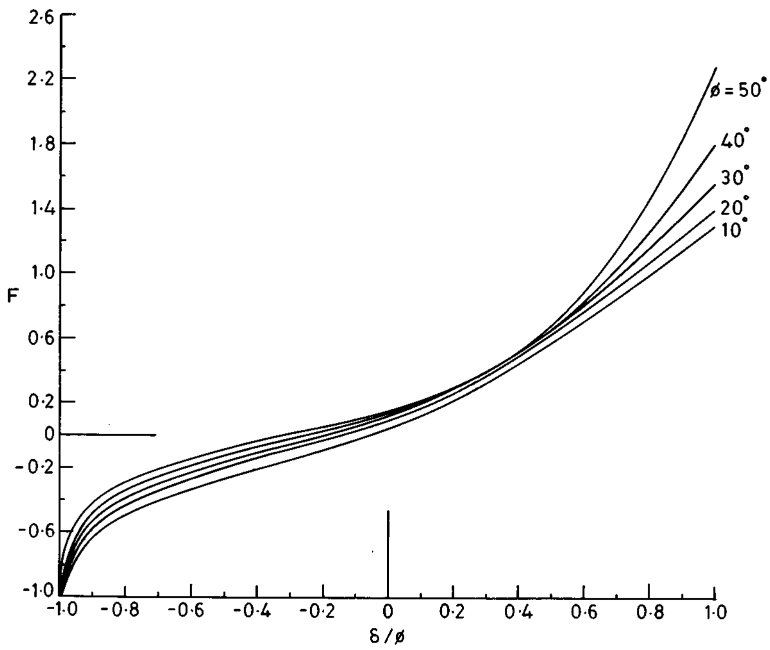

Figure 4. Variation of the factor F

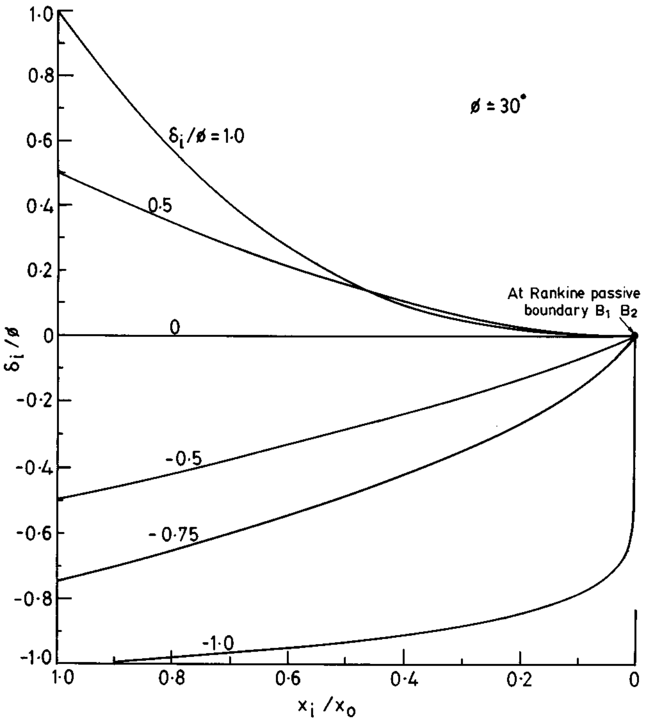

Figure 5. Distribution of the interslice shear force for $\phi = 30°$

$\phi = 30°$. For positive values of δ, the interslice shear force dissipates at much faster rate close to the retaining wall. Whereas for negative values of δ, the rate of dissipation of the interslice shear force is found to be slower near the retaining wall.

While dealing with the problem of stability of slopes, Morgenstern and Price[1] show that higher the curvature of the failure surface, greater will be the ratio between shear to the normal forces at the slice interface. In the present case, the curvature of the failure surface will be greater near the bottom of the wall for positive δ, and for negative δ the curvature values will become higher near the Rankine boundary. Accordingly, this observation justifies the above findings of the distribution of the interslice shear force by the proposed method.

Also, while determining the passive earth pressure over a vertical retaining wall in sand for positive values of δ, it was observed by Shields and Tolunay[5] that the dissipation of the interslice shear force occurs mainly close to the retaining wall. Their observations are further confirmed by the findings from the proposed method.

APPLICATION POTENTIAL FOR LAYERED SYSTEMS

Although the proposed methodology is illustrated for homogenous medium, it is possible to extend the same to layered media. Distribution of interslice forces depends on (i) location of resultant earth pressure on the wall (ii) δ_i along the wall and (iii) δ_i close to the point where the failure surface meets the ground. Since the problem is of finding earth pressure for given wall friction angle, and since δ_i near ground surface can be assessed on the assumption that state of

plastic equilibrium exists, only a reasonable assessment of point of application of resultant earth pressure at the wall needs to be made.

CONCLUSIONS

A method of slices, based on the assumption of distribution of interslice forces but satisfying all the requirements of statical equilibrium, has been developed to deal with the problem of earth pressure determination in sands. By the proposed method, the magnitudes of passive earth pressures and the variation of interslice shear forces against a vertical wall for a log-spiral composite failure surface have been examined. Due to the complete satisfaction of equilibrium conditions, the magnitudes of passive pressures become exactly the same as those obtained from Terzaghi's method. The variation of interslice shear forces is also in accordance with earlier observations.

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